Testing the Berry phase model for extraordinary Hall effect in $SrRuO_3$

Yevgeny Kats,* Isaschar Genish, and Lior Klein
Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

James W. Reiner† and M. R. Beasley
T. H. Geballe Laboratory for Advanced Materials, Stanford University, Stanford, California 94305, USA

(Dated: February 2, 2008)

Recently it has been suggested that the complicated temperature dependence of the extraordinary Hall effect (EHE) in the itinerant ferromagnet $SrRuO_3$ could be explained by the Berry phase effect in the crystal momentum space. We test this model by measurements of EHE as a function of an applied magnetic field at a constant temperature and show that the results seem to contradict the Berry phase mechanism.

PACS numbers: 75.47.-m, 72.25.Ba, 75.50.Cc, 72.15.Gd

The Hall effect in magnetic materials includes, in addition to an *ordinary* (or *regular*) Hall effect (OHE), which originates from the Lorentz force and depends on the magnetic induction **B**, an *extraordinary* (or *anomalous*) Hall effect (EHE), which depends on the magnetization **M**. Usually, the EHE is attributed to spin-dependent scattering, and the total Hall effect is given by

$$\rho_{xy} = \rho_{xy}^{OHE} + \rho_{xy}^{EHE} = R_0 B_z + R_s(\rho)\mu_0 M_z, \quad (1)$$

where R_0 is the ordinary Hall coefficient related to the carrier density n, and R_s is the extraordinary Hall coefficient, which depends on the resistivity ρ as $R_s = a\rho + b\rho^2$, where the linear term is due to a spin-dependent preferred direction in scattering ("skew scattering"),[1] and the quadratic term is due to a lateral displacement involved in the scattering ("side jump").[2]

Recently, it has been suggested that the Berry phase effect[3] in the crystal momentum space (**k** space) can also give rise to EHE.[4,5,6] This is an intrinsic effect, which does not involve scattering, but it depends on the Bloch states and their occupation. In this model, the EHE is described as

$$\rho_{xy}^{EHE} = -\rho^2 \sigma_{xy}^{BP}(M), \quad (2)$$

where the Berry phase transverse conductivity $\sigma_{xy}^{BP}(M)$ does not depend on ρ, and the dependence of σ_{xy}^{BP} on M should be calculated from the band structure. First, this mechanism was invoked to explain the EHE in (III,Mn)V ferromagnetic semiconductors,[4] then in $SrRuO_3$ (Ref. 5) (which is the subject of the current paper), and later it was shown that the Berry phase effect in **k** space can be the dominant mechanism even in iron.[6] Actually such mechanism for the EHE had been suggested by Karplus and Luttinger[7] a long time ago, but it was disregarded later. This effect should be distinguished, however, from the Berry phase effect related to a motion in a topologically nontrivial spin background in *real* space, which has been also proposed as a source of EHE for some materials.[8]

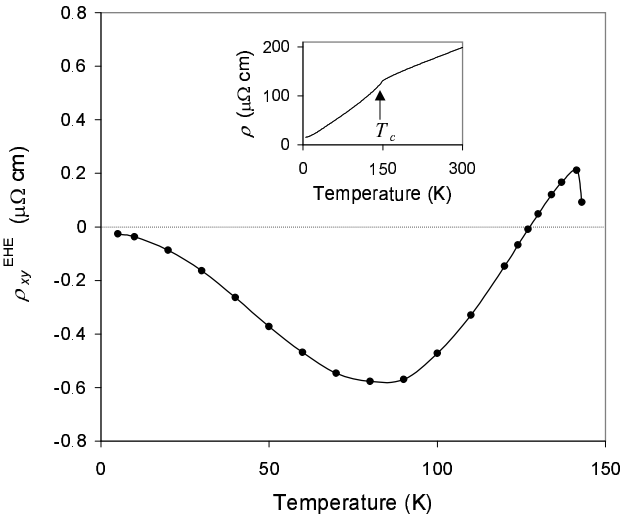

FIG. 1: Extraordinary Hall effect ρ_{xy}^{EHE} (due to the spontaneous magnetization) as a function of temperature. The inset shows the longitudinal resistivity ρ as a function of temperature.

The EHE in the 4*d* itinerant ferromagnet $SrRuO_3$ exhibits a nonmonotonic temperature dependence, including a change of sign[9,10] (see Fig. 1), and R_s does not follow the relation $R_s = a\rho + b\rho^2$.[10] Fang *et al.*[5] argued that this behavior can be explained by the Berry phase effect in **k** space, which predicts a peculiar nonmonotonic dependence of σ_{xy}^{BP} on M. The authors supported their contention by band calculations, which predicted EHE of a correct order of magnitude and roughly reproduced its temperature dependence. However, band calculations for $SrRuO_3$ are very sensitive to the input parameters.[5,12] Therefore, while the calculations support the explanation, they leave open the possibility that in practice the Berry phase effect in $SrRuO_3$ is much smaller, and the EHE is caused by a different mechanism.

Another point which raises questions regarding the applicability of the calculations of Fang *et al.* is their as-

sumption that the exchange band splitting vanishes at T_c. In many itinerant ferromagnets, and probably also in $SrRuO_3$,[13] the band splitting does not disappear at T_c, but at a temperature higher by an order of magnitude. In such materials, at T_c, the magnetization disappears on the long scale, but the short-range order remains, and the spin-split bands are quite well defined locally (see, e.g., Ref. 14). According to the calculation of Fang *et al.*, the EHE changes sign when the band splitting is about one-third of its zero-temperature value, which probably does not happen below T_c.

The experiment presented here explores the changes in EHE resulting from changes in M due to a magnetic field applied at a fixed temperature. This allows us to test the applicability of the Berry phase model directly, by a comparison between temperature-dependent and field-dependent behavior, without making assumptions regarding the details of the band structure. Particularly, we inquire whether the quantity that vanishes at $T \simeq 127$ K in Fig. 1 is the ρ-dependent R_s from Eq. (1) or the M-dependent σ_{xy}^{BP} from Eq. (2).

We study epitaxial films of $SrRuO_3$ grown by reactive electron beam coevaporation[15] on miscut ($\sim 2^\circ$) $SrTiO_3$ substrates. The films are single phase, with a single easy axis of magnetization roughly at 45° out of the plane of the film (the direction of the easy axis varies slightly as a function of temperature).[16] The film whose results are presented here has a thickness of 30 nm, and $T_c \simeq 147$ K. The films were patterned by photolithography. The current path was perpendicular to the easy axis. The residual longitudinal offset in Hall effect measurements was canceled by repeating the measurements with a reversed magnetic field and taking half the difference of the results. All measurements were performed with the films uniformly magnetized, including at zero magnetic field.

In order to separate the OHE contribution, we measured the Hall effect at a low magnetic field ($H \leq 0.4$ T) as a function of the direction of the field. In such fields, the change in ρ_{xy} is linear in H, implying that the change in ρ_{xy}^{EHE}, if it is significant, is also linear in H. In addition, for such fields **M** does not rotate away from the easy axis because the anisotropy field is of order of 10 T.[17] Since the easy axis is at $\alpha \simeq 45^\circ$ (α is defined at the right bottom part of Fig. 2), the EHE and the OHE contributions have different symmetries, and can be separated. Particularly, the EHE contribution should not be affected at all when the magnetic field is applied perpendicularly to the easy axis. In general, we expect

$$\Delta\rho_{xy} = R_0 H\cos\alpha + \frac{d\rho_{xy}^{EHE}}{dM}\chi H\cos(\alpha-\alpha_{ea}), \qquad (3)$$

where α_{ea} is the direction of the easy axis, χ is the susceptibility, and $\rho_{xy}^{EHE}(\rho, M)$ is considered to be a function of M alone, since for constant temperature and magnetization direction ρ is a function of M (Lorentz magnetoresistance is negligible at the temperature of our measurement).[11] Figure 2 shows the additional Hall effect (i.e., after subtracting the EHE measured at zero field) as a function of the direction of the field, and a fit according to Eq. (3). It turns out that the parts of the OHE and the EHE in the field-induced Hall effect are comparable in magnitude. For a field applied along the easy axis: $(60 \pm 3)\%$ of the change in Hall effect was due to the OHE, while $(40 \pm 3)\%$ was due to the EHE.

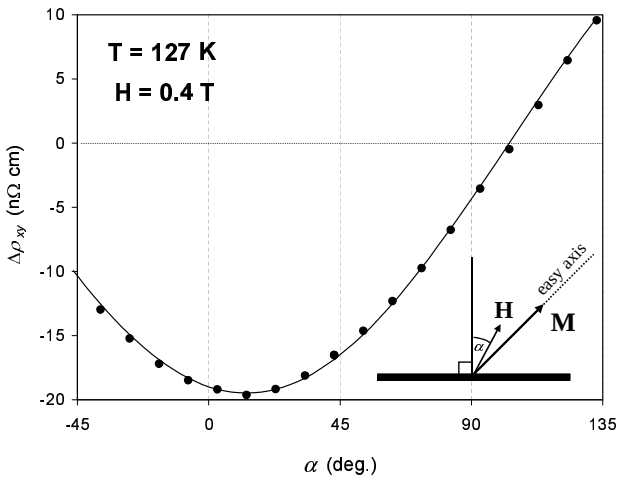

FIG. 2: Field-induced contribution to the Hall effect at $T = 127$ K, $H = 0.4$ T, as a function of the direction of the field (the angle α is shown at the right bottom part of the figure). The solid curve is a fit from which the OHE contribution was evaluated.

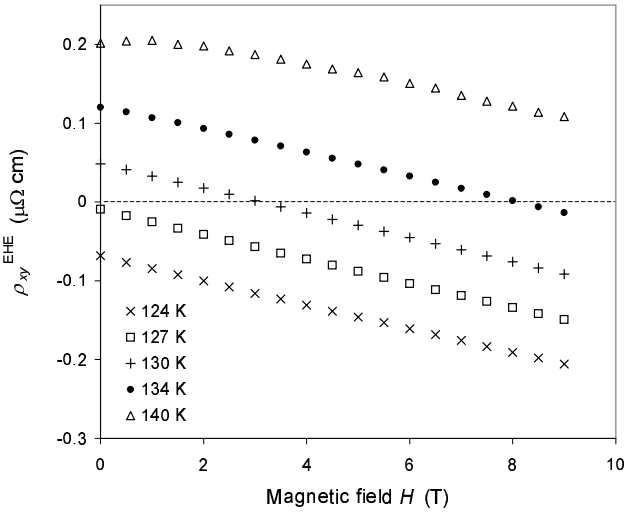

FIG. 3: Extraordinary Hall effect as a function of the magnetic field H at several temperatures (indicated in the legend).

Figure 3 shows the EHE as a function of the magnetic field H at different temperatures, after subtracting the OHE contribution. (The field was applied along the easy axis, in order to create maximal possible changes in M.) Interestingly, while the magnetization increases with the increasing field, not only does the EHE decrease, it even changes sign. Furthermore, EHE exists even at $T = 127$

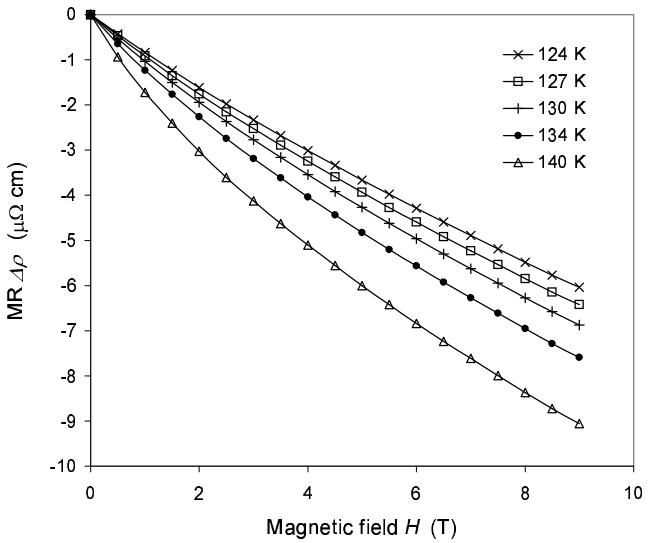

FIG. 4: Magnetoresistance $\Delta\rho(H) = \rho(H) - \rho(0)$ as a function of the magnetic field H, corresponding to the measurements presented in Fig. 3.

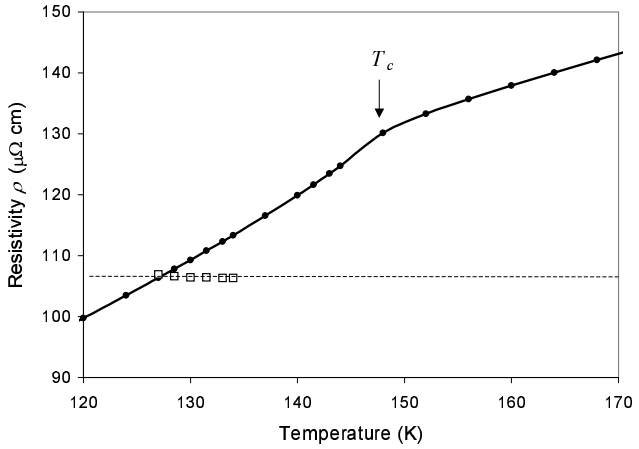

FIG. 5: The solid curve shows the temperature dependence of the zero-field resistivity ρ, and the squares denote the resistivity for which EHE vanishes at applied magnetic field, as a function of the temperature at which the field is applied.

K, where the zero-field R_s (see Fig. 1) vanishes.

These results seem to qualitatively agree with the predictions of the Berry phase model for these temperatures, since by applying a magnetic field we reach values of M that at zero field exist at lower temperatures: Figure 1 implies that in the range of temperatures presented in Fig. 3, $|\sigma_{xy}^{BP}(M)|$ decreases with increasing M; therefore, the EHE is expected to decrease when a magnetic field is applied.

On the other hand, the increase in M diminishes magnetic scattering, resulting in a negative magnetoresistance (MR) $\Delta\rho(H) = \rho(H) - \rho(0)$ (see Fig. 4).

Thus the results can qualitatively agree also with the prediction based on Eq. (1), since by applying a magnetic field we attain lower resistivities ρ, and in our range of temperatures, R_s decreases with decreasing resistivity (see Fig. 1 and inset).

Quantitative examination of the results supports the second possibility. It turns out, for example, that the MR ($\simeq -7$ $\mu\Omega$ cm) which is required at $T = 134$ K to make the EHE vanish brings the resistivity to the zero-field resistivity of $T = 127$ K (where the EHE vanishes at zero field). Figure 5 shows this pattern for a range of temperatures: the EHE always vanishes at the same value of ρ. This behavior is consistent with Eq. (1).

The vanishing of EHE at constant resistivity cannot be consistent with Eq. (2), since the identical resistivities do not correspond to identical values of M. While direct magnetic measurements could be advantageous, accurate magnetization measurements of thin films are plagued by big substrate contributions. On the other hand, it is possible to analyze the question by considering changes in magnetic scattering involved in our experiment.

From a temperature $T > 127$ K, vanishing EHE can be achieved either by lowering the temperature to 127 K or by applying an appropriate magnetic field. In both cases ρ decreases to the same value. However, in the first case, the decrease in ρ is partly related to a decrease in non-magnetic scattering (phonons, etc.), while in the second case the whole change in ρ is due to change in magnetic scattering.[11] Therefore, the magnetic scattering is different in the two cases, indicating different values of M. Thus, it is not a particular value of M in $\sigma_{xy}^{BP}(M)$, which makes EHE vanish.

Quantitatively, we estimate that the non-magnetic part of $d\rho/dT$ around 130 K is about 0.50 $\mu\Omega$ cm/K, which is the value of $d\rho/dT$ above T_c where the magnetic resistivity saturates. Therefore, non-magnetic resistivity plays an important role. For example, only 3.5 $\mu\Omega$ cm of the 7 $\mu\Omega$ cm difference in the zero-field resistivity between 134 and 127 K is due to magnetic resistivity. The magnetic resistivity of 127 K is achieved at 134 K already for $H = 3.4$ T (this is the field for which the MR is 3.5 $\mu\Omega$ cm, see Fig. 4), while the EHE vanishes only at $H = 8.1$ T.

In conclusion, when temperature-dependent and field-dependent measurements of the EHE are combined, the results cannot be simply explained in terms of the Berry phase model. On the other hand, it seems that Eq. (1) describes the EHE correctly, although the microscopic origin of the ρ-dependence of R_s remains unclear.

We appreciate valuable comments from J. S. Dodge. We acknowledge support by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities.

* Present address: Department of Physics, Harvard University, Cambridge, MA 02138.
† Present address: Department of Applied Physics, Yale University, New Haven, CT 06520-8284.

[1] J. Smit, Physica (Amsterdam) **24**, 39 (1958).
[2] L. Berger, Phys. Rev. B **2**, 4559 (1970).
[3] M. V. Berry, Proc. R. Soc. Lond. **A392**, 45 (1984).
[4] T. Jungwirth, Q. Niu, and A. H. MacDonald, Phys. Rev. Lett. **88**, 207208 (2002).
[5] Z. Fang, N. Nagaosa, K. S. Takahashi, A. Asamitsu, R. Mathieu, T. Ogasawara, H. Yamada, M. Kawasaki, Y. Tokura, and K. Terakura, Science **302**, 92 (2003).
[6] Y. Yao, L. Kleinman, A. H. MacDonald, J. Sinova, T. Jungwirth, D.-S. Wang, E. Wang, and Q. Niu, Phys. Rev. Lett. **92**, 037204 (2004).
[7] R. Karplus and J. M. Luttinger, Phys. Rev. **95**, 1154 (1954); J. M. Luttinger, *ibid.* **112**, 739 (1958).
[8] J. Ye, Y. B. Kim, A. J. Millis, B. I. Shraiman, P. Majumdar, and Z. Tesanovic, Phys. Rev. Lett. **83**, 3737 (1999); Y. Taguchi, Y. Oohara, H. Yoshizawa, N. Nagaosa, and Y. Tokura, Science **291**, 2573 (2001); R. Shindou and N. Nagaosa, Phys. Rev. Lett. **87**, 116801 (2001).
[9] M. Izumi, K. Nakazawa, Y. Bando, Y. Yoneda, and H. Terauchi, J. Phys. Soc. Jpn. **66**, 3893 (1997).
[10] L. Klein, J. W. Reiner, T. H. Geballe, M. R. Beasley, and A. Kapitulnik, Phys. Rev. B **61**, R7842 (2000).
[11] Y. Kats, L. Klein, J. W. Reiner, T. H. Geballe, M. R. Beasley, and A. Kapitulnik, Phys. Rev. B **63**, 054435 (2001).
[12] G. Santi and T. Jarlborg, J. Phys.: Condens. Matter **9**, 9563 (1997).
[13] J. S. Dodge, E. Kulatov, L. Klein, C. H. Ahn, J. W. Reiner, L. Mieville, T. H. Geballe, M. R. Beasley, A. Kapitulnik, H. Ohta, Yu. Uspenskii, and S. Halilov, Phys. Rev. B **60**, R6987 (1999).
[14] V. Korenman, J. L. Murray, and R. E. Prange, Phys. Rev. B **16**, 4032 (1977); V. Korenman and R. E. Prange, Phys. Rev. Lett. **53**, 186 (1984).
[15] S. J. Benerofe, C. H. Ahn, M. M. Wang, K. E. Kihlstrom, K. B. Do, S. B. Arnason, M. M. Fejer, T. H. Geballe, M. R. Beasley, and R. H. Hammond, J. Vac. Sci. Technol. B **12**, 1217 (1994).
[16] L. Klein, J. S. Dodge, C. H. Ahn, J. W. Reiner, L. Mieville, T. H. Geballe, M. R. Beasley, and A. Kapitulnik, J. Phys.: Condens. Matter **8**, 10111 (1996).
[17] For example, if a field of 0.4 T is applied at 60° relative to the easy axis, the magnetization will rotate by about 1°, so that the error due to incorrect modeling of the *field-induced* change in the Hall effect is only about 4% of it. The additional error due to change in the *zero-field* EHE can be made smaller than that error if the measurement is performed at a temperature where the zero-field EHE is small, as we did ($T = 127$ K).